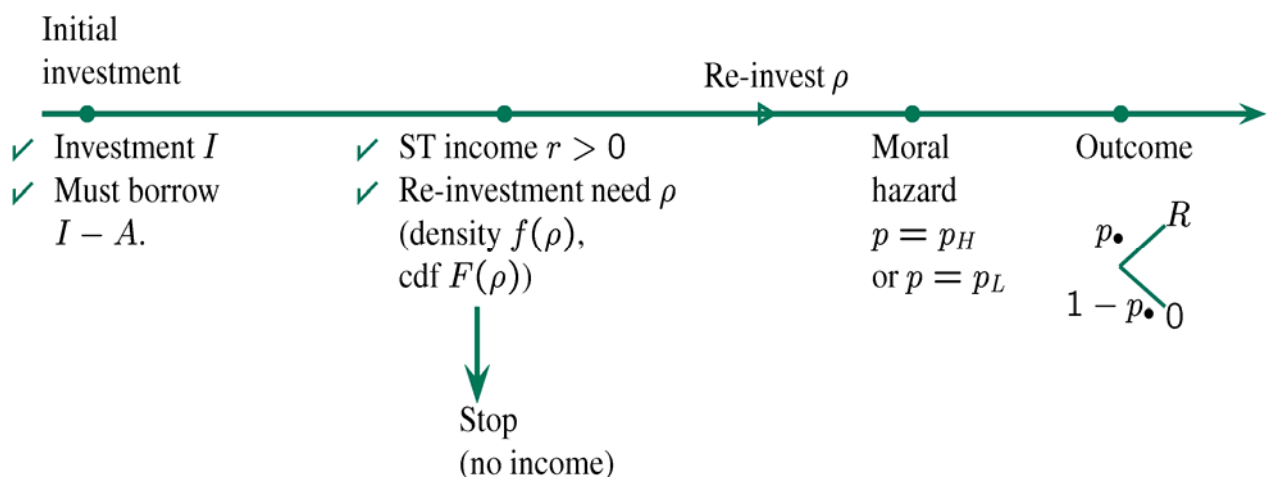


## Liquidity management

- Multistage financing
- An intermediate date between the financing stage and the realization of the project outcome.
- Following up on the discussion of the liquidity/accountability tradeoff in chapter 4.
- The borrower needs to prepare for a liquidity shock.
- The borrower should hoard reserves.
  - Holding liquid securities
  - Credit line
  - Retentions
- Hoarding of reserves is an insurance mechanism
  - True even if borrower is risk neutral
  - Value of funds higher in bad states than in good states, because of credit rationing.
  - Borrower wants to transfer wealth from good states to bad states – which is what an insurance contract does.

## Basic model

- Fixed investment, with a stochastic need for reinvestment at an intermediate date.



- Date 0: Investment  $I$ , own assets  $A$ , borrowing need  $I - A$ .
- Date 1 – the intermediate date:
  - Investment yields a short-term return  $r$ ; deterministic and verifiable.
  - Continuation requires a *reinvestment* of size  $\rho \geq 0$ , *ex ante* unknown: probability distribution  $F(\rho)$ , density  $f(\rho)$ .
  - The value of  $\rho$  becomes known at date 1.
  - No reinvestment means liquidation of the firm, liquidation value 0.
- Date 2 – in case of reinvestment at date 1: Investment returns  $R$  if success, 0 if failure. Success probability  $p$  depends on borrower's effort:  $p = p_H$  if she behaves,  $p = p_L < p_H$  if not.
- Risk neutrality. Limited liability. Competition among lenders.
- Contract:  $\{r_b, R_b, \rho^*\}$ 
  - $r_b$  and  $R_b$  – what borrower receives at dates 1 and 2.
  - $\rho^*$  – the cutoff reinvestment requirement: continue if and only if  $\rho \leq \rho^*$ .
- Borrower's net utility equals net present value of the project:
 
$$U_b(\rho^*) = [r + F(\rho^*)p_H R] - [I + \int_0^{\rho^*} \rho f(\rho) d\rho]$$
  - Second term: expected total investment
- Borrower's incentive constraint:
 
$$R_b \geq \frac{B}{\Delta p}$$
- Borrower receives 0 at date 1:  $r_b = 0$ .
  - All of  $r$  is payed out to outside investors.
  - Zero  $r_b$  increases  $R_b$  and alleviates the incentive problem at date 2.
- Expected pledgeable income:
 
$$\mathcal{R}(\rho^*) = r + F(\rho^*)p_H \left[ R - \frac{B}{\Delta p} \right] - \int_0^{\rho^*} \rho f(\rho) d\rho$$
  - Investors must cover all the reinvestment

- NPV is maximized at  $\rho^* = p_H R = \rho_1$ .
  - $U_b'(\rho^*) = f(\rho^*)p_H R - \rho^* f(\rho^*)$ .
  - For  $\rho^* < \rho_1$ , the expected gain from rescuing the project is larger than the cost.
- Pledgeable income is maximized at  $\rho^* = p_H \left[ R - \frac{B}{\Delta p} \right] = \rho_0$ .
  - For  $\rho^* > \rho_0$ , the cost to the investors from continuing is larger than what they expect to get in return.

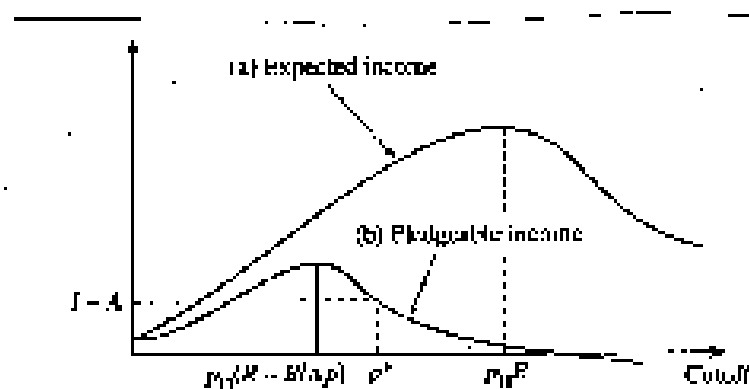


Figure 5.2, p. 204

- Three cases
  - Efficient cutoff:  $\mathcal{A}(\rho_1) \geq I - A$ .
    - The NPV-maximizing cutoff leaves enough for the investors:  $\rho^* = \rho_1$ .
  - Too much liquidation:  $\mathcal{A}(\rho_1) < I - A \leq \mathcal{A}(\rho_0)$ 
    - $r_b = 0$ ,  $R_b = B/\Delta p$ , and
 
$$\rho^* \in [\rho_0, \rho_1] \text{ solves } \mathcal{A}(\rho) = I - A$$
    - Credit rationing at date 1: In order to secure funds at date 0, the borrower accepts a reduced reinvestment cutoff at date 1.
  - No funding:  $I - A > \mathcal{A}(\rho_0)$ 
    - Even maximizing pledgeable income is not enough.

## Maturity at a cash rich firm

- *Cash rich firm*:  $r > \rho^*$ ; high short-term returns.
- Implementing the optimal contract
  - Short-term debt:  $d = r - \rho^*$ .
  - Long-term debt:  $D = R - \frac{B}{\Delta p}$  (to be paid if continuation)
    - Note: erratum for footnote 7 on p. 204.
- A theory of *maturity structure* of debt
  - Stronger firms have larger  $A$ , and subsequently (weakly) higher  $\rho^*$  and therefore less short-term debt.
  - The more current debt a firm has, the lower is its  $A$ , and the more short-term its future debt will be.
- Short-term debt vs dividend

## Credit lines for cash poor firms

- Cash poor firm:  $r < \rho^*$ . The extreme case:  $r = 0$ .
- With  $r = 0$ , there are no short-term returns to cover (in part) the liquidity needs at the intermediate date.
- Can a wait-and-see strategy work?
  - At date 1, the value of  $\rho$  is known. But the outside investors are not able to supply more funds than what the firm is worth to them, so the firm will only get funding if
$$\rho \leq p_H \left[ R - \frac{B}{\Delta p} \right] = \rho_0.$$
  - This is not optimal, since  $\rho^* \in [\rho_0, \rho_1]$ .
- It is better to *hoard reserves* at date 0 to face the liquidity shock at date 1.
  - *Liquidity management* is necessary.

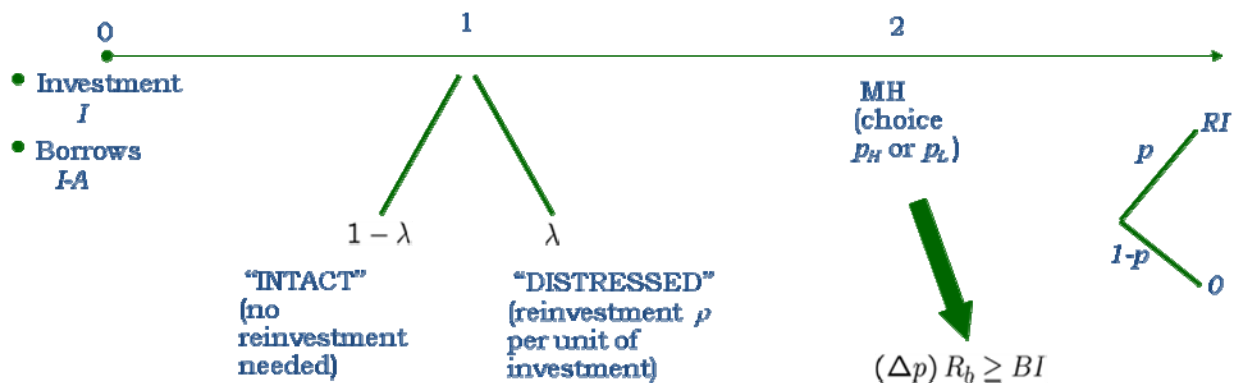
- Two ways to hoard reserves:
  - Borrowing  $I + \rho^*$  at date 0, with a covenant that no further claims be issued at date 1, so that initial claimholders are not diluted.
  - Securing a *line of credit* equal to  $\rho^* - \rho_0$ , with a right to dilute initial claimholders in order to get  $\rho_0$  in new funds at date 1.
    - A line of credit is an agreement providing credit up to a certain amount.
  - The line of credit must be *non-revokable*; otherwise, the lender would not want to abide with the agreement in cases where  $\rho \in (\rho_0, \rho^*)$ .

### Growth opportunities

- An alternative scenario: if you do not reinvest at the intermediate date, you don't have to close down; but if you do reinvest, you increase the prospects of your project.
  - Reinvestment increases probabilities of success from  $p_H$  and  $p_L$  (depending on borrower efforts) to  $p_H + \tau$  and  $p_L + \tau$ , where  $0 < \tau < 1 - p_H$ .
- Better growth opportunities (higher  $\tau$ ) call for longer maturities.

## The liquidity-scale tradeoff

- Liquidity management with a variable investment.
- The entrepreneur now faces a choice between a larger investment and more liquidity.
- Variable-investment model.
- First a simple version – two values of the per-unit liquidity shock
  - 0, with probability  $1 - \lambda$ : the firm is *intact*.
  - $\rho$ , with probability  $\lambda$ : the firm is *in distress*.



- Initial investment  $I$ . Continuation, which requires a reinvestment  $\rho I$  if the firm is in distress at date 1, is subject to moral hazard.
- Project yields  $RI$  at date 2 if success, 0 otherwise.
- Success probability  $p_H$  or  $p_L$ .
- Private benefit from misbehaving  $BI$ .
- Assumption:  $\rho_0 < c < \rho_1$ , where  $c = \min\left\{1 + \lambda\rho, \frac{1}{1 - \lambda}\right\}$ .
- Borrower receives  $R_b$  if success, 0 otherwise, where  $R_b \geq \frac{B}{\Delta p}$ .
- If distress: abandon or pursue the project?

- Abandon project
  - Investors' breakeven constraint

$$(1 - \lambda)\rho_0 I = I - A$$

- Entrepreneur's net utility = NPV

$$U_b^0 = [(1 - \lambda)\rho_1 - 1]I = \frac{(1 - \lambda)\rho_1 - 1}{1 - (1 - \lambda)\rho_0} A = \frac{\rho - \frac{1}{1 - \lambda}}{\frac{1}{1 - \lambda} - \rho_0} A$$

- Compare with case without liquidity shock:  $\lambda = 0$ .

- Pursue project
  - Investors' breakeven constraint

$$\rho_0 I = (1 + \lambda\rho)I - A$$

- Entrepreneur's net utility = NPV

$$U_b^1 = [\rho_1 - (1 + \lambda\rho)]I = \frac{\rho_1 - (1 + \lambda\rho)}{(1 + \lambda\rho) - \rho_0} A$$

- Pursuing the project in case of distress at date 1 is better than abandoning it if:

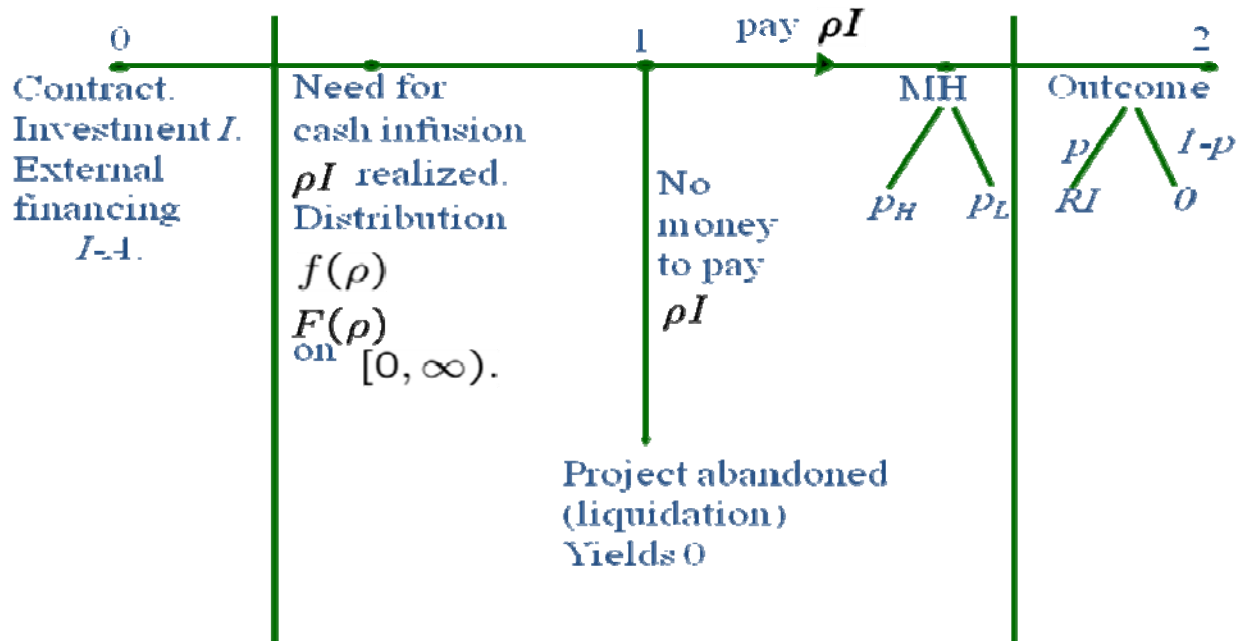
$$U_b^1 \geq U_b^0 \Leftrightarrow 1 + \lambda\rho \leq \frac{1}{1 - \lambda} \Leftrightarrow \rho \leq \frac{1}{1 - \lambda}$$

- Withstanding the liquidity shock is optimal if it is
  - low:  $\rho$  is low
  - likely:  $\lambda$  is high.

- If  $\rho_0 < \rho \leq \frac{1}{1 - \lambda}$ , then *liquidity management* is required.

## A continuum of liquidity shocks

- Continuous investment, continuous shock.
- At date 1, continuation requires a reinvestment  $\rho I$ , where  $\rho \geq 0$ .
  - Per-unit-of-investment cost overruns.
  - Probability distribution  $F(\rho)$ , density  $f(\rho)$ .



- $NPV(\tilde{\rho})$  – net present value for a given cutoff  $\tilde{\rho}$ .  

$$NPV(\tilde{\rho}) = \{F(\tilde{\rho})p_H R - [1 + \int_0^{\tilde{\rho}} \rho f(\rho) d\rho]\} I$$
- Assumption: There exists some  $\tilde{\rho}$  such that  $NPV(\tilde{\rho}) > 0$ .
- Question: What is the optimal cutoff rule  $\rho^*$ ?

- Incentive constraint if continuation:  $R_b \geq \frac{BI}{\Delta p}$

- Breakeven constraint with cutoff at  $\rho^*$ :

$$F(\rho^*)p_H(RI - R_b) \geq I - A + \int_0^{\rho^*} \rho I f(\rho) d\rho$$



- Borrowing capacity:

$$I \leq k(\rho^*)A = \frac{1}{1 + \int_0^{\rho^*} \rho f(\rho) d\rho - \rho_0 F(\rho^*)} A$$

- Recall the equity multiplier without liquidity shock:  $k = \frac{1}{1 - \rho_0}$
- Liquidity shocks reduce the equity multiplier:  $k(\rho^*) < \frac{1}{1 - \rho_0}$ .
- Due to competition among creditors, borrower obtains NPV( $\rho^*$ ).

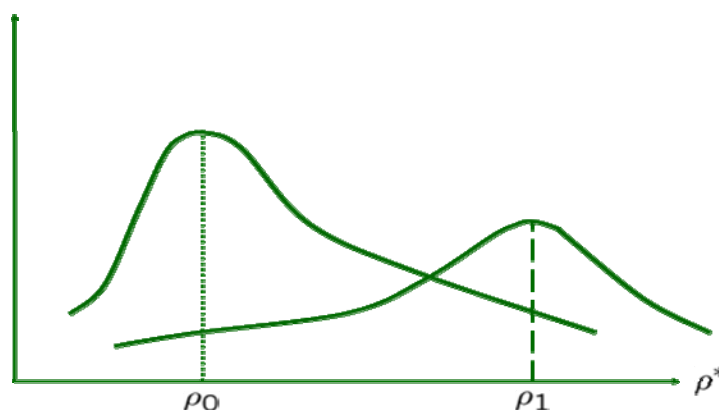
$$U_b = \{F(\rho^*)\rho_1 - [1 + \int_0^{\rho^*} \rho f(\rho) d\rho]\} I \Leftrightarrow$$

$$U_b = m(\rho^*)k(\rho^*)A,$$

where

$$m(\rho^*) = F(\rho^*)\rho_1 - 1 - \int_0^{\rho^*} \rho f(\rho) d\rho$$

- The *margin* per unit of investment:  $m(\rho^*)$
- The borrower must trade off the margin and the equity multiplier
  - Maximizing  $m(\rho^*)$  would maximize profit and yield  $\rho^* = \rho_1$ . But  $k'(\rho_1) < 0$ .
  - Maximizing  $k(\rho^*)$  would maximize pledgeable income and yield  $\rho_0$ . But  $m'(\rho_0) > 0$ .



- Write the borrower's net utility as

$$U_b = \frac{\rho_1 - c(\rho^*)}{c(\rho^*) - \rho_0} A, \text{ where: } c(\rho^*) = \frac{1 + \int_0^{\rho^*} \rho f(\rho) d\rho}{F(\rho^*)}$$

- Note:  $F(\rho^*)c(\rho^*) = 1 + \int_0^{\rho^*} \rho f(\rho) d\rho$ 
  - $c(\rho^*)$  is the *expected cost per unit of effective investment*
- Maximizing  $U_b$  is tantamount to minimizing  $c(\rho^*)$ .

- Minimizing  $c(\rho^*)$ :

$$c'(\rho^*) = \frac{\rho^* f(\rho^*)F(\rho^*) - [1 + \int_0^{\rho^*} \rho f(\rho) d\rho] f(\rho^*)}{[F(\rho^*)]^2}$$

$$c'(\rho^*) = \frac{f(\rho^*)}{F(\rho^*)} [\rho^* - c(\rho^*)].$$

- The optimal cutoff is implicitly defined by:  $\rho^* = c(\rho^*)$ .
- In equilibrium, the borrower's net utility is:

$$U_b = \frac{\rho_1 - \rho^*}{\rho^* - \rho_0} A$$

- The optimum cutoff lies between the expected per-unit-of-investment pledgeable income and income:

$$\rho_0 < \rho^* < \rho_1$$

- *Trading off size and liquidity*: Increasing the cutoff above  $\rho^*$  would be good for profit but would also increase the demand for liquidity.

### Risk management

- Suppose there is some residual uncertainty  $\varepsilon$  in the reinvestment requirement at date 1, such that  $E(\varepsilon | \rho) = 0$ .
- Consequences are adverse if liquidity falls short of a reinvestment
- Calls for buying insurance even if the entrepreneur is risk neutral.

## Endogenous liquidity shocks

- The entrepreneur may incur efforts to reduce – or even eliminate – the need for reinvestments. How to provide her with incentives to do this?
- A simple situation:
  - Before date 1, the borrower can incur effort costs  $c$  that will eliminate reinvestment needs completely:  $\rho = 0$  with probability 1. If not, then  $\rho$  is drawn from the distribution  $F(\rho)$  as before.
  - If the firm is cash poor – little or no income  $r$  at date 1 – the optimal contract has a covenant that no more funds shall be reinvested. But is this credible?
  - If the borrower does *not* incur costs  $c$  and the liquidity needs turn out to be  $0 \leq \rho \leq \rho_0$ , then it is in both lender's and borrower's interest to renegotiate the original contract.
  - This scope for renegotiation reduces the borrower's incentives to incur the effort costs  $c$ .
  - *Soft budget constraint.*
- More generally: Suppose the borrower can act at date 0 in a way that would improve the project, and that information arrives at date 1 that indicates whether or not she did so.
  - Moral hazard at both dates 0 and 1 (with respect to outcomes at dates 1 and 2).
  - Examples
    - Short-term income  $r$  stochastic *and* dependent on date-0 efforts
    - The project, if abandoned at date 1, has a liquidation value  $L$  that is stochastic and dependent on date 0 efforts
    - The project's date-2 return can be improved through efforts at date 0, and information about these improvements may be available before the reinvestment decision is made.
- Here: short-term income affected stochastically by date-0 efforts.

## Endogenous intermediate income

- Variable-investment model.
- An investment of  $I$  at date 0 returns  $rI$  at date 0, where  $r \in [0, r^+]$ , is verifiable. (In addition, there is also the usual stochastic return  $RI$  at date 2, subject to date-1 moral hazard.)
- Exerting effort affects the probability distribution of  $r$ .
- If the entrepreneur works at date 0, then  $r$  is distributed according to  $G(r)$ , with density  $g(r)$ . If the entrepreneur shirks at date 0, then  $r$  is distributed according to  $\tilde{G}(r)$ , with density  $\tilde{g}(r)$ .
- The likelihood ratio

$$l(r) = \frac{g(r) - \tilde{g}(r)}{g(r)}$$

- *The monotone likelihood ratio property* (MLRP):  $l'(r) \geq 0$ .
  - Implying that the distribution of  $r$  improves if the entrepreneur works:  $G(r) \leq \tilde{G}(r), \forall r$ .
- Private benefit at date 0 if entrepreneur shirks:  $B_0I$ .
- Benchmark: Credibility is not an issue – the “no soft budget constraint” (NSBC) case.
- Contract:  $\{\rho^*(r), \Delta(r)\}$ , where
  - $\rho^*(r)$  is the state-contingent cutoff
  - $\Delta(r) \geq 0$  is the borrower’s state-contingent “extra rent” per unit of investment:

- If continuation,

$$\Delta(r) = p_H \left( R_b - \frac{BI}{\Delta p} \right),$$

what the borrower receives over and above the minimum required to preserve date-1 incentives.

- If liquidation,  $\Delta(r)$  is cash compensation.

- Lenders' breakeven constraint ( $IR_l$ ):

$$\left\{ \int_0^{r^+} \left[ r + F(\rho^*(r))\rho_0 - \int_0^{\rho^*(r)} \rho f(\rho) d\rho - \Delta(r) \right] g(r) dr \right\} I \geq I - A$$

- Borrower's date-0 incentive constraint ( $IC_b$ ):

$$\left\{ \int_0^{r^+} \left[ F(\rho^*(r))(\rho_1 - \rho_0) + \Delta(r) \right] [g(r) - \tilde{g}(r)] dr \right\} I \geq B_0 I \Leftrightarrow$$

$$\left\{ \int_0^{r^+} \left[ F(\rho^*(r))(\rho_1 - \rho_0) + \Delta(r) \right] l(r) g(r) dr \right\} I \geq B_0 I$$

- The optimal contract maximizes borrower's net utility subject to the two above constraints, with respect to  $\{\rho^*(r), \Delta(r), I\}$ . We ignore the choice of  $I$  for the moment.

$$U_b = \left\{ \int_0^{r^+} \left[ r + F(\rho^*(r))\rho_1 - \int_0^{\rho^*(r)} \rho f(\rho) d\rho - 1 \right] g(r) dr \right\} I$$

- Lagrangian multipliers:  $\mu$  for  $IR_l$  and  $\nu$  for  $IC_b$ .
- Pointwise maximization.

○ For each  $r$ , find the optimal pair  $\{\rho^*(r), \Delta(r)\}$

- Fix  $r$ . First-order conditions with respect to  $\rho^*(r)$  and  $\Delta(r)$ :

$$\{f(\rho^*)\rho_1 - \rho^*f(\rho^*) + \mu[f(\rho^*)\rho_0 - \rho^*f(\rho^*)] + \nu[f(\rho^*)(\rho_1 - \rho_0)]l(r)\} \\ \times g(r)I = 0$$

$$\{-\mu + \nu l(r)\}g(r)I = 0$$

$\Leftrightarrow$

$$\rho^*(r) = \frac{\rho_1 + \mu\rho_0}{1 + \mu} + \frac{\nu(\rho_1 - \rho_0)}{1 + \mu}l(r)$$

$$\mu = \nu l(r)$$

○ But the constraint  $\Delta(r) \geq 0$  may be binding. Therefore,

- either:  $\Delta(r) > 0 \Rightarrow \mu = \nu l(r) \Rightarrow \rho^* = \rho_1$ ,
- or:  $\Delta(r) = 0 \Rightarrow -\mu + \nu l(r) \leq 0 \Rightarrow \rho^* \leq \rho_1$ .

- Note:  $E_{G(\cdot)}[l(r)] = \int_0^{r^+} \frac{g(r) - \tilde{g}(r)}{g(r)} g(r) dr = \int_0^{r^+} g(r) dr - \int_0^{r^+} \tilde{g}(r) dr = 0$

- This implies:  $E[\rho^*(r)] = \frac{\rho_1 + \mu\rho_0}{1 + \mu}$

- In expectation, the cutoff is a weighted average of  $\rho_1$  and  $\rho_0$ , and  $\rho_1 < E[\rho^*(r)] < \rho_0$ ; as in the case without date-0 moral hazard, the firm *trades off size and liquidity*.

- We can write:

$$\rho^*(r) = E[\rho^*(r)] + \lambda l(r),$$

$$\text{where: } \lambda = \frac{\nu}{1 + \mu} (\rho_1 - \rho_0) > 0.$$

- By assumption (MLRP):  $l'(r) \geq 0$ . Therefore:  $\frac{d\rho^*}{dr} \geq 0$ .

- The continuation rule is more lenient, the higher is the date-1 income  $r$ .

- Two possibilities:

- $\rho^*(r)$  increases moderately

- because the date-0 incentive problem is small

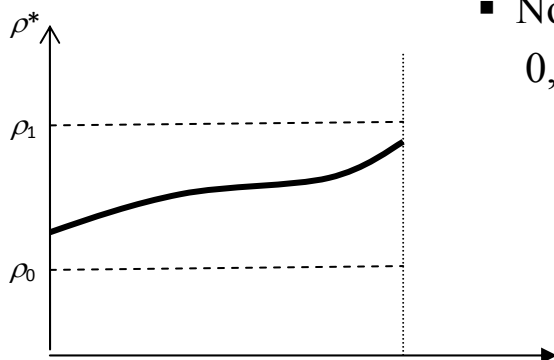
- date-0 private benefits  $B_0$  not very high, so that the borrower's date-0 incentive constraint is not very restrictive, making  $\nu$  low;

- date-0 liquidity shocks being mainly outside the borrower's control, so that  $l(r)$  stays close to 0.

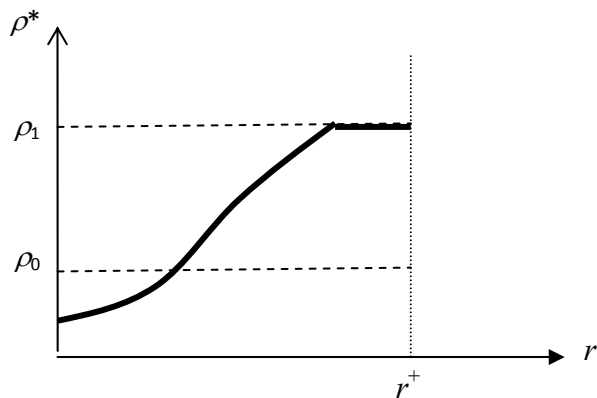
- or because the date-1 incentive problem is small

- date-1 private benefits  $B$  small, or  $\Delta p/p_H$  large, again making  $\nu$  low.

- No extra rent to the borrower:  $\Delta(r) = 0, \forall r$ .

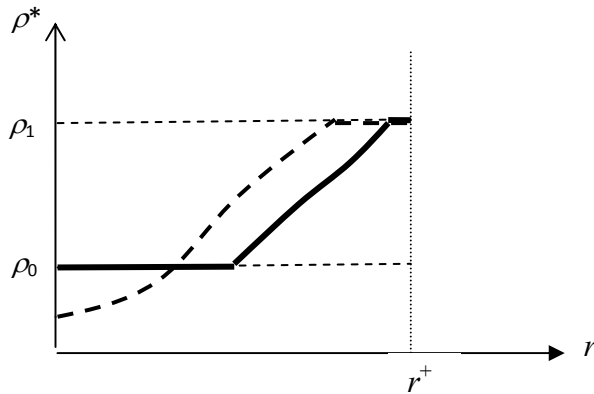


- $\rho^*(r)$  increases steeply
  - because one or both of the two moral hazard problems are more serious
  - When intermediate income is high, first-best can be reached:  $\rho^* = \rho_1$ .
  - Extra rent to the borrower at high  $r$ : When intermediate income is high, she gets to keep some of it.
  - At a low intermediate income, we may even have  $\rho^* < \rho_0$ .



- *Soft budget constraint*:  $\rho^* < \rho_0$  is not credible.
  - The parties will renegotiate a contract whenever  $r$  is realized and  $\rho^*(r) < \rho_0$ .
  - Formally, same problem as before, with an added constraint:  $\rho^* \geq \rho_0$ .
  - When incentive problems are small, so that there is only a moderate increase in  $\rho^*(r)$  in the NSBC case, there is no change in the optimal contract.
  - When incentive problems are greater, the constraint  $\rho^* \geq \rho_0$  binds for small values of  $r$ .

- Increasing  $\rho^*$  in order to satisfy the credibility constraint at low values of  $r$  calls for decreasing it for higher values of  $r$ , in order to keep satisfying the lenders' breakeven constraint.



- Credibility problems at low values of  $r$  decreases continuation – and reduces efficiency – at larger values.



## Free cash flow

- If the firm has more cash than it needs, there are incentives for *overinvestment*. It has been argued that debt may mitigate this problem.
- Back to the discussion of the liquidity-scale tradeoff.
- But now there is a deterministic short-term income  $rI$ , which is fully pledgeable.
- Lenders' breakeven constraint with cutoff at  $\rho^*$ :

$$rI + F(\rho^*)p_H(RI - R_b) \geq I - A + \int_0^{\rho^*} \rho f(\rho) d\rho$$

- Everything as if the unit investment cost is  $(1 - r)$  rather than 1.
- Cutoff implicitly given by:

$$\rho^* = c(\rho^*) = \frac{1 - r + \int_0^{\rho^*} \rho f(\rho) d\rho}{F(\rho^*)}$$

- Cutoff  $\rho^*$  is now *decreasing* in the short-term income  $r$ .
  - A high  $r$  makes it possible to reduce continuation in order to increase the borrowing capacity.
- The *free-cash-flow* assumption:  $r > \rho^*$ .
  - The entrepreneur would like to commit herself not to reinvest the amount  $(r - \rho^*)I$ .
  - This calls for *short-term debt*, that is, debt to be payed at the intermediate date.
  - In more general settings, short-term debt may not fully resolve the free-cash-flow problem.